Eur. Phys. J. B **58**, 51–53 (2007)
DOI: 10.1140/epjb/e2007-00194-3

THE EUROPEAN PHYSICAL JOURNAL B

Comment on "A local realist model for correlations of the singlet state" by K. De Raedt, K. Keimpema, H. De Raedt, K. Michielsen and S. Miyashita

M.P. Seevinck[1,a] and J.-Å. Larsson[2,b]

[1] Institute for History and Foundations of Science, Utrecht University, P.O Box 80.000, 3508 TA Utrecht, The Netherlands
[2] Matematiska Institutionen, Linköpings Universitet, SE-581 83 Linköping, Sweden

Received 14 March 2007
Published online 28 July 2007 –

Abstract. De Raedt et al. [Eur. Phys. J. B **53**, 139 (2006)] have claimed to provide a local realist model for correlations of the singlet state in the familiar Einstein-Podolsky-Rosen-Bohm (EPRB) experiment when time-coincidence is used to decide which detection events should count in the analysis, and furthermore that this suggests that it is possible to construct local realistic models that can reproduce the quantum mechanical expectation values. In this letter we show that these conclusions cannot be upheld since their model exploits the so-called coincidence-time loophole. When this is properly taken into account no startling conclusions can be drawn about local realist modelling of quantum mechanics.

PACS. 03.65.Ud Entanglement and quantum nonlocality – 03.65.Ta Foundations of quantum mechanics

De Raedt et al. [1] have recently claimed to have constructed a local realist model for correlations of the singlet state, in which time-coincidence is used to decide which detection events should count in the analysis. Furthermore, they claim that their model maximally violates the well-known Clauser-Horne-Shimony-Holt (CHSH) inequality [2], and conclude that their work "suggests that it is possible to construct event-based simulation models that satisfy Einstein's criteria of local causality and realism and can reproduce the expectation values calculated by quantum theory" [1].

Here, we will put the model used by De Raedt et al. in its proper context and show that, although the model gives the sinusoidal correlations familiar from quantum mechanics, the conclusions drawn by De Raedt et al. cannot be upheld. This is because their model is based on post-selection, something which is known to enable quantum-like correlations from a local realist model. This possibility was first noted by Pearle in the late sixties [3] and has received quite some interest in the years that followed [4,5] and has been especially active more recently [6–12, ...]; a full list of references would be immense. We will see below that the model in its general form exploits the so-called "coincidence-time loophole" [13], and that the usual CHSH inequality is inappropriate for this situation because of the postselection. The appropriately modified inequality is not violated by the De Raedt et al. model, even though it gives quantum-like correlations. Notable is also that the post-selection procedure for coincidence in time used by De Raedt et al. is not used in all experimental realizations of the EPRB experiment [14,15] and consequently their model cannot reproduce all experimental realizations of the EPRB experiment (see e.g., [16]). Since this is the case, we would argue that no startling conclusion can be drawn from the model about local realist modelling of quantum-mechanical singlet correlations, nor about local realist modelling of quantum mechanics in general.

Let us first go through some notation. The setup of the EPRB experiment has two measurement stations $i = 1, 2$ with Stern-Gerlach magnets that can be set to measurement directions $\mathbf{a}_1$ and $\mathbf{a}_2$ respectively, and the angular difference between these settings is denoted α. The local hidden variable in the model of De Raedt et al. is denoted $S_{n,i}$ (event number n, station i) and is a direction in space. From this hidden variable, the model gives results $x_{n,i}$ and detection times $t_{n,i}$ with time resolution τ. Coincidences only occur when the detection times are within a time window of width W, i.e., when

$$|t_{n,1} - t_{n,2}| \leq W. \tag{1}$$

[a] e-mail: seevinck@phys.uu.nl
[b] e-mail: jalar@mai.liu.se

These coincidences are used to calculate the correlations of the outcomes $x_{n,i}$ for different setting combinations (see Eq. ([1]:3)). This is exactly the same kind of modeling as that discussed in [13].

In the limit where $W = \tau \to 0$ (i.e., where the time window and the time-tag resolution both go to zero) De Raedt et al. obtain the sinusoidal correlation of the singlet state from their model. They then argue that this correlation violates the well-known CHSH inequality

$$\big|E(\mathbf{a},\mathbf{c}) - E(\mathbf{a},\mathbf{d}) + E(\mathbf{b},\mathbf{c}) + E(\mathbf{b},\mathbf{d})\big| \le 2, \tag{2}$$

where $E(\mathbf{a},\mathbf{b})$ is the correlation between outcomes for settings $\mathbf{a}$ and $\mathbf{b}$, etc. They furthermore claim that the maximal quantum violation is obtained using their model (i.e., where the left-hand side of (2) is $2\sqrt{2}$). However, inequality (2) is not valid for their model; the correlations they calculate (that reach $2\sqrt{2}$) are not of the form of the ones on the left-hand side of (2).

The problem is the postselection of events that are close enough in time for which "[...] the simultaneity of two detection events will depend on *both settings*, even though the underlying physical processes that control this are completely local" [13]. A postselection procedure of this kind invalidates the original CHSH inequality (2) since the ensemble on which the correlations are evaluated changes with the settings, while (2) requires them to remain the same; see [6,13]. The correlation calculated in [1] is not $E(\mathbf{a}_1,\mathbf{a}_2)$, as was claimed, but

$$E(\mathbf{a}_1,\mathbf{a}_2|\Lambda_{\mathbf{a}_1\mathbf{a}_2}) = -\mathbf{a}_1\cdot\mathbf{a}_2, \tag{3}$$

where $E(\mathbf{a}_1,\mathbf{a}_2|\Lambda_{\mathbf{a}_1\mathbf{a}_2})$ is the *conditional* correlation, conditioned on a coincidence for the settings $\mathbf{a}_1$ and $\mathbf{a}_2$. Consequently, inequality (2) cannot be used. The appropriate inequality includes this conditioning on coincidence and reads [13]

$$\begin{aligned}\big|E(\mathbf{a},\mathbf{c}|\Lambda_{\mathbf{ac}}) &- E(\mathbf{a},\mathbf{d}|\Lambda_{\mathbf{ad}})\\ &+ E(\mathbf{b},\mathbf{c}|\Lambda_{\mathbf{bc}}) + E(\mathbf{b},\mathbf{d}|\Lambda_{\mathbf{bd}})\big| \le \frac{6}{\gamma} - 4.\end{aligned} \tag{4}$$

The quantity γ is the probability of coincidence. Quantum mechanical correlations that violate the CHSH inequality (2) by the value $2\sqrt{2}$ will violate (4) only if $\gamma > \gamma_0 = 3 - 3/\sqrt{2} \approx 0.8787$; this bound is necessary and sufficient (see [13] for further details). That is, if $\gamma \le \gamma_0$, it is possible to construct a local realist model that gives a value of $2\sqrt{2}$ on the left-hand side. Such a model is given in [13] which furthermore saturates the bound.

Let us now go back to the model of De Raedt et al. If $W = \tau \ll 1$, we have

$$\gamma \le 8\tau\cot\frac{\alpha}{2}, \qquad \alpha \ne 0, \tag{5a}$$

$$\gamma \lesssim 6\pi\tau^{2/3}, \qquad \alpha = 0, \tag{5b}$$

(see Appendix A) so evidently the value of γ approaches zero when $W = \tau \to 0$. This is below the bound specified above and, although the model gives sinusoidal correlations — and may be interesting as such — it does not violate the relevant Bell inequality (4)[1]. For other local realist models with a sinusoidal interference pattern, but with a nonzero probability of coincidence see, e.g., references [7,8].

In conclusion, even though the model by De Raedt et al. [1] does give (conditional) correlations as strong as quantum mechanics, it does not model the singlet state as such, because in the model the probability of coincidence must go to zero to obtain the sinusoidal interference pattern. This means that the model does not violate the relevant Bell inequality, because it is far below the relevant coincidence-probability bound $\gamma_0 \approx 0.8787$. In addition, this is far below the coincidence probabilities of previously published local realist models [7,8]. Finally, even though De Raedt et al. claim their model can reproduce the coincidences of recent experimental results, it cannot: optical experiments reach $\gamma \approx 0.05$ [17] and ion-trap experiments even reach $\gamma = 1$ [16]; the latter consequently does not fall prey to the coincidence-time loophole[2]. This reinforces the conclusion drawn in [13] of the importance of eliminating postselection in future EPRB experiments, and, as we've seen here, postselection must be duly accounted for in any local realist modelling of them.

Appendix A

The probability of coincidences γ of the model in [1] is given by the denominator of ([1]:6) and can be calculated using the density of coincidences

$$P(T_1,T_2,W) \le \tau\frac{\min(T_1,T_2)}{T_1T_2}. \tag{6}$$

The above bound is valid when $W = \tau$ and is given in [1]. One should also remember that

$$P(T_1,T_2,W) \le 1, \tag{7}$$

since it is the probability that a coincidence occurs given the values of the hidden variables $S_{n,i}$ and the settings $\mathbf{a}_i$. When $\tau \ll 1$ and $\alpha \ne 0$ (i.e., $\mathbf{a}_1 \ne \mathbf{a}_2$) the inequality (7) is automatically satisfied when (6) is, and

$$\begin{aligned}\gamma &= \int_0^{\pi}\int_0^{2\pi} P(T_1,T_2,W)\sin\theta d\theta d\phi\\ &\le \tau\int_0^{\pi}\int_0^{2\pi}\frac{\min(T_1,T_2)}{T_1T_2}\sin\theta d\theta d\phi\\ &= 2\tau\int_0^{2\pi}\frac{\min\left(\sin^2\phi,\sin^2(\phi-\alpha)\right)}{\sin^2\phi\sin^2(\phi-\alpha)}d\phi = 8\tau\cot\frac{\alpha}{2}.\end{aligned} \tag{8}$$

[1] Actually, since De Raedt et al. use $W = \tau$ (time window length equals time resolution), they are in effect using the more well-studied "efficiency loophole" [3,5,6], but that is perhaps more of a technical side note.

[2] Nor to the detection loophole.

For the case when $\alpha = 0$ (i.e., $\mathbf{a}_1 = \mathbf{a}_2$ and $T_1 = T_2$), the requirement (7) needs to be taken into account, and

$$\begin{aligned}
\gamma &= \int_0^{\pi}\int_0^{2\pi} P(T_1, T_2, W)\sin\theta d\theta d\phi \\
&\leq \int_0^{\pi}\int_0^{2\pi} \min\left(\frac{\tau}{T_i}, 1\right)\sin\theta d\theta d\phi \\
&= \int_0^{\pi}\int_0^{2\pi} \min\left(\frac{\tau}{(1-\cos^2\phi\sin^2\theta)^{3/2}}, 1\right)\sin\theta d\theta d\phi \\
&= 4\pi\left(\tau^{2/3}\sqrt{1-\tau^{2/3}} + \frac{\tau^{2/3}}{1+\sqrt{1-\tau^{2/3}}}\right) \\
&\approx 6\pi\tau^{2/3}. \qquad (9)
\end{aligned}$$

The last approximation is good when $\tau \ll 1$.

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